Date 28/6/18 UNIT-IV

Unit-IV Interpolation

*** Introduction:**

Since $y = f(x)$ be the given function. The given function defined in the interval (a, b) then it is called "interpolation".

Consider 'x' takes the values $x_0, x_1, x_2, x_3, x_4 \ldots, x_n$ the corresponding y-values are $y_0, y_1, y_2, y_3, y_4 \ldots, y_n$ respectively. And the differences of x are is 'h' then

$x_1 - x_0 = h$, $x_2 - x_1 = h$, $x_3 - x_2 = h, \ldots, x_n - x_{n-1} = h$

$\Rightarrow x_1 = x_0 + h$.

$\Rightarrow x_2 = x_1 + h \Rightarrow x_2 = (x_0 + h) + h$

$$\boxed{x_2 = x_0 + 2h}$$

$\Rightarrow x_3 = x_2 + h \Rightarrow x_3 = (x_0 + 2h) + h$

$$\boxed{x_3 = x_0 + 3h}$$

$\Rightarrow x_n - x_{n-1} = h \Rightarrow \boxed{x_n = x_0 + nh}$

Given, $y = f(x)$

$y_0 = f(x_0)$

$y_1 = f(x_1)$

$y_1 = f(x_0 + h)$

$y_2 = f(x_2)$

$= f(x_0 + 2h)$

$y_3 = f(x_3)$

$= f(x_0 + 3h)$

$y_n = f(x_n)$

$$\boxed{y_n = f(x_0 + nh)}$$

The differences

$y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ are represented by

$\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_3, \ldots$ respectively are called first order forward differences and Δ is called forward difference operator.

The differences

$\Delta y_1 - \Delta y_0, \Delta y_2 - \Delta y_1, \Delta y_3 - \Delta y_2, \ldots$ are represented by $\Delta^2 y_0, \Delta^2 y_1, \Delta^2 y_2, \ldots$ are called second order "forward differences"

The differences

$\Delta^2 y_1 - \Delta^2 y_0, \Delta^2 y_2 - \Delta^2 y_1, \Delta^2 y_3 - \Delta^2 y_2, \ldots$ are represented by $\Delta^3 y_0, \Delta^3 y_1, \Delta^3 y_2, \ldots$ respectively are called third order forward differences.

The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ are represented by $\nabla y_1, \nabla y_2, \nabla y_3, \nabla y_4, \ldots$ respectively are called first order backward differences and '∇' is called Backward difference operator.

The differences $\nabla y_2 - \nabla y_1, \nabla y_3 - \nabla y_2, \nabla y_4 - \nabla y_3, \ldots$ are represented by $\nabla^2 y_2, \nabla^2 y_3, \nabla^2 y_4, \ldots$ respectevely are called second order backward differences.

The differences $\nabla^2 y_3 - \nabla^2 y_2, \nabla^2 y_4 - \nabla^2 y_3, \ldots$ are represented by $\Delta^3 y_3, \Delta^3 y_4, \nabla^3 y_5, \ldots$ respectively are called third order backward differences.

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The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$, are represented by small (δ) $\delta y_{1/2}, \delta y_{3/2}, \delta y_{5/2}, \delta y_{7/2} \ldots$ respectively are called central differences and 'δ' is called central difference operator.

The differences $\delta y_{3/2} - \delta y_{1/2}, \delta y_{5/2} - \delta y_{3/2}, \delta y_{7/2} - \delta y_{5/2} \ldots$ are represented by $\delta^2 y_1, \delta^2 y_2, \delta^2 y_3, \ldots$ respectively are called second order central differences

Similarly $\delta^2 y_2 - \delta^2 y_1, \delta^2 y_3 - \delta^2 y_2, \delta y_4^2 - \delta_3^2 \ldots \delta^2 \ldots$ are represented by $\delta^3 y_{3/2}, \delta^3 y_{5/2}, \delta^3 y_{7/2} \ldots$ respectively are called the third order central differences

Shifting Operator

Since 'E' is called shifting operator. It shifts the given function into the next level.

Cons Therefore

$Ey_0 = y_1 \Rightarrow Ef(x_0) = f(x_1)$

$$\boxed{Ef(x_0) = f(x_0+h)}$$

$Ey_1 = y_2 \Rightarrow Ef(x_1) = f(x_2)$

$E \cdot f(x_0+h) = f(x_0+2h)$

$E \cdot Ef(x_0) = f(x_0+2h)$

$$\boxed{E^2 f(x_0) = f(x_0+2h)}$$

$\therefore E^n f(x_0) = f(x_0+nh)$

Similarly $E^3 f(x_0) = f(x_0+3h)$

Therefore $\boxed{E^n f(x) = f(x+nh)}$ **

Note

Since $E^n f(x) = f(x+nh)$

put $n = -n \Rightarrow E^{-n} f(x) = f(x+(-n)h)$

*** $\boxed{E^{-n} f(x) = f(x-nh)}$

Book Work

Since we know the $y_1 - y_0 = \Delta y_0 \rightarrow ①$

and $Ey_0 = y_1 \rightarrow ②$

from ① & ②

$$Ey_0 - y_0 = \Delta y_0$$

$$(E-1)\cancel{y_0} = \Delta \cancel{y_0}$$

$$E - 1 = \Delta$$

$$\boxed{E = 1 + \Delta}$$

Relation between S.O and forward differences

Since we know that $y_1 - y_0 = \nabla y_1 \rightarrow ①$

we know and $Ey_0 = y_1$

$$\Rightarrow y_0 = E^{-1} y_1 \rightarrow ②$$

from ① & ②

$$y_1 - E^{-1} y_1 = \nabla y_1$$

$$y_1 (1 - E^{-1}) = \nabla y_1$$

$$1 - E^{-1} = \nabla$$

$$\boxed{E^{-1} = 1 - \nabla}$$

Relation between shifting operator and backward differences

Since we know that

$$y_1 - y_0 = \delta y_{1/2} \rightarrow$$

$$\Rightarrow y_{\frac{1}{2}+\frac{1}{2}} - y_{\frac{1}{2}-\frac{1}{2}} = \delta y_{1/2}$$

$\because$ $E^1 y_3 = y_{3+1}$

$E^1 y_0 = y_{0+1}$

$$E^{1/2} y_{1/2} - E^{-1/2} y_{1/2} = \delta y_{1/2}$$

$$\cancel{y_{1/2}} \left[E^{1/2} - E^{-1/2} \right] = \delta \cancel{y_{1/2}}$$

$$\boxed{E^{1/2} - E^{-1/2} = \delta}$$

Relation between central difference and shifting operator

Average Operator

'μ' is called Average operator. such that

$$\mu y_n = \frac{y_{n+1/2} + y_{n-\frac{1}{2}}}{2}$$

$$\mu y_n = \frac{E^{1/2} y_n + E^{-1/2} y_n}{2}$$

$$\mu \cancel{y_n} = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right] \cancel{y_n}$$

$$\boxed{\mu = \frac{E^{1/2} + E^{-1/2}}{2}}$$

The above equation is the relation between Average operator and shifting operator

Pascal's Triangle.

$$\begin{array}{ccccccccccc} & & & & & 0 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

$$\Delta^4 y_0 = 1y_1 - 4y_2 + 6y_3 - 4y_4 + 1y_5$$

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Newtons Forward interpolation formulae

Consider $y = f(x)$ be the given function. x creates the values, $x_0, x_1, x_2, \ldots x_n$ and the common difference between 'x' is 'h'.

The corresponding 'y' values are $y_0, y_1, y_2, \ldots y_n$ respectively then

$$y_n = f(x_0 + nh)$$
$$= E^n f(x_0)$$

$$(1+\Delta)^n y_0 = y_n$$

$$\because (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots$$

$$y_n = (1+\Delta)^n = \left[1+n\Delta+\frac{n(n-1)}{2!}\Delta^2+\frac{n(n-1)(n-2)}{3!}\Delta^3+\cdots\right]y_0$$

$$y_n = y_0+n\Delta y_0+\frac{n(n-1)}{2!}\Delta^2 y_0+\frac{n(n+1)(n-2)}{3!}\Delta^3 y_0+\cdots$$

Newton's Backward Interpolation Formulae

At arbitrary value $x=x_n$ the corresponding y values is y_n then $y_n = f(x_n)$

$$\Rightarrow y_n = f(x_n+nh)$$
$$= E^n f(x_n)$$
$$= (E^{-1})^{-n} f(x_n)$$
$$= (1-\nabla)^{-n} y_n$$

$$\because (1-x)^{-n} = \left[1+nx+\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\frac{n(n+1)(n+2)(n+3)}{4!}\right.$$

$$y_n = (1-\nabla)^{-n} = \left[1+n\nabla+\frac{n(n+1)}{2!}\nabla^2+\frac{n(n+1)(n+2)}{3!}\nabla^3+\frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4+\right.$$

$$y_n = y_n+n\nabla y_n+\frac{n(n+1)}{2!}\nabla^2 y_n+\frac{n(n+1)(n+2)}{3!}\nabla^3 y_n+\frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$

Problems

1. find $\Delta f(x)$, $f(x) = x^3-x^2+x+10$, $h=1$

Solu) Since we know that

$$\Delta f(x) = f(x+h)-f(x)$$
$$= f(x+1)-f(x)$$
$$= (x+1)^3-(x+1)^2+(x+1)+10-[x^3-x^2+x+10]$$
$$= x^3+1^3+3x^2 1+3x-[x^2+1+2x]+x+1$$
$$+10-x^3+x^2-x-10$$
$$= \cancel{x^3}+1+3x^2+3x-\cancel{x^2}-\cancel{1}-2x+\cancel{x}+\cancel{1}+\cancel{10}$$
$$-\cancel{x^3}+\cancel{x^2}-\cancel{x}-\cancel{10}$$
$$= 3x^2+3x+1-2x$$

$\therefore \Delta f(x) = 3x^2 + x + 1$

2. find $\Delta^2 f(x)$, given $f(x) = e^{2x}$, $h = 1$

solu) Since $\Delta f(x) = f(x+h) - f(x)$

we know that

$\Delta f(x) = f(x+1) - f(x)$

$= e^{2(x+1)} - e^{2x}$

$= e^{2x+2} - e^{2x}$

$= e^{2x} \cdot e^2 - e^{2x}$

$\Delta f(x) = e^{2x}(e^2 - 1)$

$\Delta e^{2x} = e^{2x}(e^2 - 1) \rightarrow (1)$

$\Delta^2 f(x) = \Delta[\Delta f(x)]$

$= \Delta[e^{2x}(e^2-1)]$

$= (e^2-1)[\Delta e^{2x}]$

$= (e^2-1)[e^{2x}(e^2-1)]$ from (1)

$\therefore \Delta^2 f(x) = (e^2-1)^2 e^{2x}$

3. If $f(x) = \frac{10}{x!}$ find $\Delta f(x)$ and $h = 1$

solu) $\Delta f(x) = f(x+h) - f(x)$

$= f(x+1) - f(x)$

$= \frac{10}{(x+1)!} - \frac{10}{x!} \Rightarrow \frac{10}{(x+1)!\,x!} - \frac{10}{x!}$

$= \frac{10 - 10(x+1)}{(x+1)!\,x!}$

$= \frac{10[1-x-1]}{(x+1)!}$

$= \frac{-10x}{(x+1)!}$

7 Show that $\delta^2 E = \Delta^2$

Solu) $\delta^2 E = \Delta^2$

$\delta = E^{1/2} - E^{-1/2}$

$\Delta = E - 1$

$L.H.S = (E^{1/2} - E^{-1/2})^2 E$

$= \left((E^{1/2})^2 + (E^{-1/2})^2 - 2E^{1/2}E^{-1/2}\right) E$

$= [E + E^{-1} - 2] E$

$= E^2 + E^{-1} \cdot E - 2E$

$= [E^2 + 1 - 2E \cdot 1]$

$= [E-1]^2$

$= \Delta^2 = R.H.S$

$L.H.S = R.H.S$

Hence proved.

8 Show that $\mu\delta = \dfrac{E - E^{-1}}{2}$

Solu) $\mu = \dfrac{E^{1/2} + E^{-1/2}}{2}$ $\delta = E^{1/2} - E^{-1/2}$

$L.H.S = \left[\dfrac{E^{1/2} + E^{-1/2}}{2}\right]\left[E^{1/2} - E^{-1/2}\right]$

$= \dfrac{(E^{1/2})^2 - (E^{-1/2})^2}{2}$

$= \dfrac{E^1 - E^{-1}}{2}$

$= R.H.S$

$E = 1+\Delta$

$E^{-1} = 1-\nabla$

$\mu = \dfrac{E^{1/2}+E^{-1/2}}{2}$

$\delta = E^{1/2} - E^{-1/2}$

9 Show that $\Delta = \nabla(1-\nabla)^{-1}$

Solu) $\Delta = \nabla(1-\nabla)^{-1}$

$1 - \nabla = E^{-1}$

$R.H.S = \nabla(E^{-1})^{-1}$

$= \nabla E,\ \nabla = 1 - E^{-1}$

$= (1-E^{-1})E$

$= E - E^{-1} \cdot E$

$= E - 1$

$= \Delta$

$=$ R.H.S

10. Write forward difference table for

x :	10	20	30	40
y :	1.1	2.0	4.4	7.9

Solu: Forward Difference Table

x	y	Δ	Δ^2	Δ^3
10	1.1			
		$=2.0-1.1$ $=0.9$		
20	2.0		$=2.4-0.9$ $=1.5$	
		$=4.4-2.0=2.4$		$=1.1-1.5$ $=-0.4$
30	4.4		$=3.5-2.4$ $=1.1$	
		$=7.9-4.4$ $=3.5$		
40	7.9			

why we took upto 3

11. Construct the difference table for the given data and evaluate $\Delta^2 f(2)$

x :	0	1	2	3	4
$f(x)$:	1.0	1.5	2.2	3.1	4.6

Solu: Difference table

x	$f(x)$	1st	2nd	3rd	4th
0	1.0				
		$1.5-1.0$ $=0.5$			
1	1.5		$0.7-0.5$ $=0.2$		
		$2.2-1.5$ $=0.7$		$0.2-0.2$ $=0.0$	
2	2.2		$0.9-0.7$ $=0.2$		$0.4-0.0$ $=0.4$
		$3.1-2.2$ $=0.9$		$0.6-0.2$ $=0.4$	
3	3.1		$1.5-0.9$ $=0.6$		
		$4.6-3.1$ $=1.5$			
4	4.6				

In the above question Δ is given so that why we should do forward

From the difference table $\Delta^2 f(2) = 0.6$

Forward starts with y_0

 Note in backward starts $y_0\ y_1\ y_2\ y_3$

From the difference table $\nabla^2 f(2) = 0.2$

12. Find the missing value of the following data.

x	1	2	3	4	5
$f(x)$	7	-	13	21	37

Solu) Difference table

x	$f(x)$	1^{st}	2^{nd}	3^{rd}	4^{th}
1	7				
		$y-7$			
2	y		$(13-y)-(y-7) = 20-2y$		
		$13-y$		$y-5-20+2y = 3y-25$	
3	13		$8-(13-y) = y-5$		$13-y-3y+25 = 38-4y$
		$21-13$ 8		$8-y+5 = 13-y$	
4	21		$16-8 = 8$		
		$37-21$ 16			
5	37				

$[(13-y)(y-7)$
$13y - y^2 + 7y - 91$
$6y - y^2 - 91$
$y^2 - 6y + 91]$

From the Difference table

$38 - 4y = 0$
$38 = 4y$
$y = 9.5$

$4)38(9.5$
36
20

$13 \times 7 = 91$

13. Prove that $u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$

Solu)

$$R.H.S = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$$
$$= u_3 + \Delta u_2 + \Delta^2 u_1 + (\Delta^2 u_2 - \Delta^2 u_1)$$
$$= u_3 + \Delta u_2 + \cancel{\Delta^2 u_1} + \Delta^2 u_2 - \cancel{\Delta^2 u_1}$$
$$= u_3 + \Delta u_2 + \Delta^2 u_2$$
$$= u_3 + \cancel{\Delta u_2} + (\Delta u_3 - \cancel{\Delta u_2})$$
$$= u_3 + \Delta u_3$$
$$= \cancel{u_3} + u_4 - \cancel{u_3}$$
$$= u_4 = P = L.H.S$$

14. Evaluate $u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

Solu)

$$[= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$$
$$= u_0 + 4(u_1 - u_0) + 6(\Delta u_0 - \Delta u_{-1}) + 10(\Delta^2 u_0 - \Delta u_{-1})$$
$$= u_0 + 4u_1 - 4u_0 + 6\Delta u_0 - 6\Delta u_{-1} + 10\Delta^2 u_0 - 10\Delta u^2_{-1}$$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 6\Delta u_{-1} - 10\Delta u_{-1}$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 10\Delta u^2_{-1} - 6\Delta u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10[\Delta^2 u_0 - \Delta^2 u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^2 u_0 - 10\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4(\Delta u_0 - \Delta u_{-1})$

$= u_0 + 4\cancel{\Delta u_0} + 10\Delta^2 u_0 - 4\cancel{\Delta u_0} + 4\Delta u_{-1}$

$= u_0 + 10\cdot\Delta^2 u_0 + 4\Delta u_{-1}$

$= u_0 + 10[\Delta u_1 - \Delta u_0) + 4(u_0 - u_{-1})$

$= u_0 + 10\Delta u_1 - 10\Delta u_0 + 4u_0 - 4u_{-1}$

$= u_0 + 10[u_2 - u_1] - 10[u_1 - u_0] + 4u_0 - 4u_{-1}$

$= 10u_2 - 20u_1 + 15u_0 - 4u_{-1}$

15. Evaluate $\Delta(e^{ax}\log(bx))$

solu) $\Delta f(x) = f(x+h) - f(x)$

$= e^{a(x+h)}\log b(x+h) - e^{ax}\log(bx)$

16. u_x is a function of x for which 5^{th} differences are constant and $u_1 + u_7 = -786$; $u_2 + u_6 = 686$; $u_3 + u_5 = 1088$ find u_4

solu) Since Given that 5^{th} differences are constants

$\therefore \Delta^6 u_1 = 0$.

Since we know that $\Delta = E - 1$ why we don't $\Delta (\Delta e)$ here

$\therefore (E-1)^6 u_1 = 0$

$[1.E^6 - 6c_1 E^5 + 6c_2 E^4 - 6c_3 E^3 + 6c_4 E^2 - 6c_5 E + 6c_6 1] u_1 = 0$

$E^6 u_1 - 6E u_1^5 + \frac{6\times5}{1\times2} E u_1^4 - \frac{6\times5\times4}{1\times2\times3} E u_1^3 + \frac{6\times5\times4\times3}{1\times2\times3\times4} E u_1^2 - \frac{6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5} E u_1 + \frac{6}{6} u_1 = 0$

$E^6 u_1 - 6E^5 u_1 + 15E^4 u_1 - 20E^3 u_1 + 15E^2 u_1 - 6E u_1 + u_1 = 0$

$u_7 - 6u_6 + 15u_5 - 20u_4 + 15u_3 - 6u_2 + u_1 = 0$

$(u_7 + u_1) - 6(u_6 + u_2) + 15(u_5 + u_3) - 20u_4 = 0$

$-786 - 6(686) + 15(1088) - 20u_4 = 0$

$-786 - 4116 + 16320 - 20u_4 = 0$

$-4902 + 16320 = 20u_4$

$20u_4 = 11418$

$\therefore u_4 = \frac{11418}{20}$

$u_4 = 570.9$

Note:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots +$$

From what to understand

Since we know that $Ef(x) = f(x+h)$ -- by Taylor's series formula

$$= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f''''(x) - - .$$

$$= f(x) + h\frac{d}{dx} f(x) + \frac{h^2}{2!}\frac{d^2}{dx^2} f(x) + \frac{h^3}{3!}\frac{d^3}{dx^3} f(x) + \frac{h^4}{4!}\frac{d^4}{dx^4} f(x) + \cdots$$

$$= f(x)\left[1 + h\frac{d}{dx} + \frac{h^2}{2!}\frac{d^2}{dx^2} + \frac{h^3}{3!}\frac{d^3}{dx^3} + \frac{h^4}{4!}\frac{d^4}{dx^4} + \cdots\right]$$

$$= f(x)\left[1 + hD + \frac{h^2}{2!}D^2 + \frac{h^3}{3!}D^3 + \frac{h^4}{4!}D^4 \cdots\right]$$

$$= f(x)\left[1 + hD + \frac{(hD)^2}{2!} + \frac{(hD)^3}{3!} + \frac{(hD)^4}{4!} + \cdots\right]$$

$\left[\because \text{Here } \frac{d}{dx} = D\right]$

$\therefore Ef(x) = f(x) \cdot e^{hD}$

$\boxed{\therefore E = e^{hD}}$ (or) $E = 1 + \Delta \Rightarrow 1 + \Delta = e^{hD}$

$\Delta = e^{hD} - 1$

17. Show that $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n!\, h^n}{x(x+h)(x+2h) \cdots (x+nh)}$

solu $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n!\, h^n}{x(x+h)(x+2h) \cdots (x+nh)}$

$\therefore \Delta f(x) = f(x+h) - f(x)$.

Now $n = 1$

$$\Delta\left[\frac{1}{x}\right] = \frac{1}{x+h} - \frac{1}{x}$$

$$= \frac{x - (x+h)}{x(x+h)}$$

$$= \frac{x - x - h}{x(x+h)}$$

$$= \frac{(-1)h}{x(x+h)} \longrightarrow (1)$$

$n = 2$

$$\Delta^2\left[\frac{1}{x}\right] = \Delta\left[\Delta\left[\frac{1}{x}\right]\right]$$

$$= \Delta\left[\frac{(-1)h}{x(x+h)}\right]$$

$$= h(-1)\left[\Delta \frac{1}{x(x+h)}\right]$$

$$= h(-1)\left[\frac{1}{(x+h)(x+2h)} - \frac{1}{x(x+h)}\right]$$

$$= (-1)h\left[\frac{x-(x+2h)}{x(x+h)(x+2h)}\right]$$

$$= (-1)h\left[\frac{\cancel{x}-\cancel{x}-2h}{x(x+h)(x+2h)}\right]$$

$$= \frac{(-1)^2\, 2\, h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2\, 2.1\, h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2\, 2!\, h^2}{x(x+h)(x+2h)} \longrightarrow (2)$$

If $n=3$

$$\Delta^3\left[\frac{1}{x}\right] = \Delta\left[\Delta^2\left[\frac{1}{x}\right]\right]$$

$$= \Delta\left[\frac{(-1)^2\, 2!\, h^2}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2\, 2!\, h^2\left[\Delta\left(\frac{1}{x(x+h)(x+2h)}\right)\right]$$

$$= (-1)^2\, 2!\, h^2\left[\frac{1}{(x+h)(x+h+h)(x+h+2h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2\, 2!\, h^2\left[\frac{1}{(x+h)(x+2h)(x+3h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2\, 2!\, h^2\left[\frac{x-(x+3h)}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= (-1)^2\, 2!\, h^2\left[\frac{\cancel{x}-\cancel{x}-3h}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= \frac{(-1)^2\, 2!\, h^2\, (-1)\, 3h}{x(x+h)(x+2h)(x+3h)}$$

$$= \frac{(-1)^3\, 1\times 2\times 3\, h^3}{x(x+h)(x+2h)(x+3h)}$$

$$\therefore \Delta^3\left[\frac{1}{x}\right] = \frac{(-1)^3\, 3!\, h^3}{x(x+h)(x+2h)(x+3h)} \longrightarrow (3)$$

Hence from (1), (2) & (3)

$$\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n\, n!\, h^n}{x(x+h)(x+2h)\cdots(x+nh)}$$

Date 5/8/18

Given, $u_0+u_8 = 1.9243$, $u_1+u_7 = 1.9590$, $u_2+u_6 = 1.9823$, $u_3+u_5 = 1.9956$ then find u_4.

18\. solu: Since $\Delta^8 u_0 = 0$

$(E-1)^8 u_0 = 0$

$u_0[1\cdot E^8 - 8C_1 E^7 + 8C_2 E^6 + 8C_3 E^5 + 8C_4 E^4 + 8C_3 E^3 + 8C_6 E^2 + 8C_7 E + 8C_8] = 0$

$u_0 E^8 - 8E^7 u_0 + \frac{8\times7}{1\times2} E^6 u_0 + \frac{8\times7\times6}{1\times2\times3} E^5 u_0 + \frac{8\times7\times6\times5}{1\times2\times3\times4} E^4 u_0 + \frac{8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6} E^2 u_0 + \frac{8\times7\times6\times5\times4\times3\times2}{1\times2\times3\times4\times5\times6\times7} E u_0$

$+ \frac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5} E^3 u_0$

$+ 8u_0 = 0$

$u_8 - 8u_7 + 28u_6 - 56u_5 + 70u_4 - 56u_3 + 28u_2 - 8u_1 + u_0 = 0$

$70u_4 + (u_0+u_8) - 8(u_1+u_7) + 28(u_2+u_6) - 56(u_3+u_5) = 0$

$70u_4 + 1.9243 - 8(1.9590) + 28(1.9823) - 56(1.9956) = 0$

$70u_4 + 1.9243 - 8(1.9590) + 55.5044 - 111.7536 = 0$ $\quad \frac{1.9243}{5}$

$70u_4 + 1.9243 - 15.672 + 55.5044$

$69.9969 = 70u_4$

$u_4 = \frac{69.9969}{70}$

$u_4 = 0.999955714$

$\therefore u_4 = 1$

19\. Find the missing term in the following

x:	0	5	10	15	20	25
y:	6 (y_0)	10 (y_1)	— (y_2)	17 (y_3)	— (y_4)	31 (y_5)

solu: Consider $\Delta^4 y_0 = 0 \rightarrow (1)$ and $\Delta^4 y_1 = 0 \rightarrow (2)$ we know that

$\Delta = E-1$ sub in (1) & (2)

$(E-1)^4 y_0 = 0$ and $(E-1)^4 y_1 = 0$

$\Rightarrow [1\cdot E^4 - 4C_1 E^3 + 4C_2 E^2 + 4C_3 E + 4C_4] y_0 = 0$ &

$[1\cdot E^4 - 4C_1 E^3 + 4C_2 E^2 + 4C_3 E + 4C_4] y_1 = 0$

$[1\cdot E^4 - 4C_1 E^3 + 4C_2 E^2 + 4C_3 E + 4C_4] y_0 = 0$ &]

$\Rightarrow [u_4 - 4u_3 + \frac{4\times3}{1\times2} u_2 + \frac{4\times3\times2}{1\times2\times3} u_1 + 4] y_0 = 0$

$\Rightarrow E^4 y_0 - 4E^3 y_0 + \frac{4\times3}{1\times2} y_0 E^2 - \frac{4\times3\times2}{1\times2\times3} E y_0 + 1\cdot y_0 = 0$ &

$E^4 y_1 - 4E^3 y_1 + \frac{4\times3}{1\times2} y_1 E^2 - \frac{4\times3\times2}{1\times2\times3} E y_1 + 1\cdot y_1 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0 \rightarrow (3)$

$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0 \rightarrow (4)$

From (3)

$[17 - 4y_3 + 6(10) - 4(6) +]$

$\Rightarrow y_4 - 4(17) + 6y_2 - 4(10) + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 68 - 40 + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 102 = 0$

$\Rightarrow y_4 + 6y_2 = 102 \rightarrow (5)$

From (4)

$31 - 4y_4 + 6(17) - 4y_2 + 10 = 0$

$31 + 102 + 10 = y_4 + 4y_2$

$143 = 4y_4 + 4y_2 \rightarrow (6)$

$$\left.\begin{aligned} 6y_2 + y_4 - 102 &= 0 \\ 4y_2 + 4y_4 - 143 &= 0 \end{aligned}\right\} \text{By } 2312$$

$\therefore y_2 = 13.25, \quad y_4 = 22.5$

$20)265(13.25$
20
65
60
50

102
4
408

858
408
450

y_2		y_4	1
1	-102	6	1
4	-143	4	4

$\dfrac{y_2}{-143+408} = \dfrac{y_4}{-408+858} = \dfrac{1}{24-4}$

$\dfrac{y_2}{265} = \dfrac{y_4}{450} = \dfrac{1}{20}$

$y_2 = \dfrac{265}{20} = 13.25 \;;\; y_4 = \dfrac{450}{20} = 22.5$

408
143
265

20: Find the missing value of the following table

x	1	2	3	4	5
y	7 (y_0)	x (y_1)	13 (y_2)	21 (y_3)	37 (y_4)

Solu) $\Delta^4 y_0 = 0$ since we know that

$\therefore E = 1 + \Delta$

$\Delta = E - 1$

$(E-1)^4 y_0 = 0$

$[1 \cdot E^4 - 4c_1 E^3 + 4c_2 E^2 - 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + \frac{4 \times 3}{1 \times 2} E^2 y_0 - \frac{4 \times 3 \times 2}{1 \times 2 \times 3} E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$37 - 4(21) + 6(13) - 4x + 4(7) = 0$

$37 - 84 + 68 - 4x + 28 = 0$

$65 + 68 - 84 = 4x$

$3 - 81 = 4x$

$-78 = 4x$

$7 - 4x =$

$7 = 4x$

$x = \frac{7}{4}$

37
28
65

$4)7(1.7$
4
30
28

84
54
30

$37 - 4\times21 + 6\times13 - 4y_1 + 7 = 0$

$-84 \quad -4y_1 + 37 + 7 + 78 = 0$

$-4y_1 + 38 = 0$

$4y_1 = 38$

$y_1 = 9.5$

21. Estimate the missing term in the following table.

x	1	2	3	4	5	6	7
y	2	4	8	—	32	64	128
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Solu) Since we know that

$\Delta^6 y_0 = 0$ $\quad E = 1+\Delta$

$\Delta = E-1$

$(E-1)^6 y_0 = 0$

$[E^6 \cdot 1 - 6c_1 E^5 + 6c_2 E^4 - 6c_3 E^3 + 6c_4 E^2 - 6c_5 E + 6c_6] y_0 = 0$

$E^6 y_0 - 6E^5 y_0 + 15E^4 y_0 - \frac{6.5.4}{1.2.3} E^3 y_0 + \frac{6.5.4.3}{1.2.3.4} E^2 y_0 - \frac{6.5.4.3.2}{1.2.3.4.5} E y_0 + y_0$

$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$

$128 - 6(64) + 15(32) - 20(y_3) + 15(8) - 6(4) + 2 = 0$

$128 - 384 + 480 - 20y_3 + 120 - 24 + 2 = 0$ $\quad 322 = 20y_3$

$730 - 284 - 24 - 20y_3 = 0$ $\quad y_3 = \frac{322}{20}$

$730 - 308 = 20y_3$

$422 = 20y_3$ $\quad y_3 = 16.1$

$y_3 = 21.1$

284, 24, 1, 305

480, 120, 128, 112, 730, 308, 422

15, 4, 120, 321, 15, 160, 32, 48

22. Given $\log 100 = 2$; $\log 101 = 2.0043$; $\log 103 = 2.0128$; $\log 104 = 2.0170$ and find $\log 102$.

Solu) Here given

x	100	101	102	103	104
$y = \log x$	2	2.0043	—	2.0128	2.0170
	y_0	y_1	y_2	y_3	y_4

Since we know that

$\Delta^4 y_0 = 0$

$\Delta = E-1$

From $E = 1+\Delta$

$(E-1)^4 y_0 = 0$

$[1.E^4 - 4c_1 E^3 + 4c_2 E^2 - 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + \frac{4\times3}{1\times2} E^2 y_0 - \frac{4\times3\times2}{1\times2\times3} E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$2.0170 - 4(2.0128) + 6y_2 - 4(2.0043) + 2 = 0$

$2.0170 - 8.0512 + 6y_2 - 8.0172 + 2 = 0$

$4.0170 - 16.0684 + 6y_2 = 0$

$6y_2 = \frac{12.0514}{6}$

$\therefore y_2 = 2.0086$

$\therefore \log_{10} 2 = 2.0086$

$2.0170 + 2 = 4.0170$

$8.0512 + 8.0172 = 16.0684$

23. Find the missing values of the following.

x	10	15	20	25	30	35
y	43 (y_0)	– (y_1)	29 (y_2)	32 (y_3)	– (y_4)	77 (y_5)

Solu) Since we know that

$\Delta^4 y_0 = 0 \;; \quad E = 1+\Delta \Rightarrow \Delta = E-1$

$(E-1)^4 y_0 = 0$

$[1.E^4 - 4c_1 E^3 + 4c_2 E^2 + 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$y_4 - 4(32) + 6(29) - 4y_1 + 43 = 0$

$y_4 - 128 + 174 - 4y_1 + 43 = 0$

$y_4 - 4y_1 = 128 - 174 - 43$

$y_4 - 4y_1 = 128 - 217$

$y_4 - 4y_1 = -89 \rightarrow (1)$ (or) $4y_1 - y_4 = 89 \rightarrow (1)$

$-(-y_4 + 4y_1) = -89$

$4y_1 - y_4 = 89$

$174 + 43 = 217$

$217 - 128 = 89$

$\Delta^4 y_1 = 0$

$(E-1)^4 y_1 = 0$

$[1.E^4 - 4c_1 E^3 + 4c_2 E^2 - 4c_3 E + 4c_4] y_1 = 0$

$E^4 y_1 - 4E^3 y_1 + 6E^2 y_1 - 4E y_1 + y_1 = 0$

$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$

$$77 - 4y_4 + 6(32) - 4(29) + y_1 = 0$$

$$y_1 - 4y_4 + 77 + 192 - 116 = 0$$

$$y_1 - 4y_4 = 116 - 192 - 77$$

$$y_1 - 4y_4 = 116 - 269$$

$$y_1 - 4y_4 = -153 \rightarrow (2)$$

192
77
269
116
153

$y_1 \quad y_4 \quad 1$

$-1 \quad -89 \quad 4 \quad -1$

$-4 \quad 153 \quad 1 \quad -4$

356 + 153 = 509 ; 89 + 612 = 1502

$$\frac{y_1}{-153-356} = \frac{y_4}{-89-612} = \frac{1}{-16+1}$$

$$\frac{y_1}{-509} = \frac{y_4}{-701} = \frac{1}{-15}$$

$$y_1 = \frac{-509}{-15} \; ; \; y_4 = \frac{-701}{-15}$$

$$y_1 = 33.9334 \; ; \; y_4 = 46.7334$$

Date 6/8/18

24. Estimate the production for 1964 and 1968 from the following data.

years (x)	1961	1962	1963	1964	1965	1966	1967
production (y)	200	220	260	—	350	—	430
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

solu) Given that years x = 1961, 1962, 1963, 1964, 1965, 1966, 1967

y = 200, 220 260 — 350 — 430

$$\Delta^5 y_0 = 0 \rightarrow (1) \qquad E = 1 + \Delta$$

$$(E-1)^5 y_0 = 0$$

$$[1 \cdot E^5 - 5c_1 E^4 + 5c_2 E^3 - 5c_3 E^2 + 5c_4 E - 5c_5] y_0 = 0$$

$$y_0 E^5 - 5 y_0 E^4 + 5c_2 E^3 y_0 - 5c_3 E^2 y_0 + 5c_4 E y_0 - 5c_5 y_0$$

$$y_5 - 5y_4 + \frac{5 \times 4}{1 \times 2} y_3 - \frac{5 \times 4 \times 3}{1 \times 2 \times 3} y_2 + \frac{5 \times 4 \times 3 \times 2}{1 \cdot 2 \cdot 3 \cdot 4} y_1 + y_0 = 0$$

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 + y_0 = 0$$

$$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) + 200 = 0$$

$$y_5 - 1750 + 10y_3 - 2600 + 1100 + 200 = 0$$

$$y_5 + 10y_3 - 3050 = 0 \rightarrow (2)$$

13050
200
12850

$\Delta^5 y_1 = 0 \rightarrow ③$

$y_6 \cdot (E-1)^5 y_1 = 0$

$[1 \cdot E^5 - 5c_1 E^4 + 5c_2 E^3 - 5c_3 E^2 + 5c_4 E - 5c_5] y_1 = 0$

$E^5 y_1 - 5c_1 E^4 y_1 + 5c_2 E^3 y_1 - 5c_3 E^2 y_1 + 5c_4 E y_1 - 5c_5 y_1 = 0$

$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - y_1 = 0$

$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$

$430 - 5y_5 + 3500 - 10y_3 + 1300 - 220 = 0$

$5010 - 5y_5 - 10y_3 = 0$

$5y_5 + 10y_3 = 5010 \rightarrow ④$

From ② and ④

$$\begin{aligned} y_5 + 10y_3 &= 3450 \\ 5y_5 + 10y_3 &= 5010 \\ \hline -4y_5 &= -1560 \end{aligned}$$

$y_5 = \cancel{\dfrac{1560}{4}}^{390} = 390$

$y_5 + 10y_3 = 3450$

$390 + 10y_3 = 3450$

$10y_3 = 3450 - 390$

$10y_3 = 3060$

$y_3 = 306$

From ① & ②,

$$\begin{array}{cccc} & y_3 & y_5 & 1 \\ 6 & -706 & 20 & 6 \\ 15 & -1196 & 15 & 15 \end{array}$$

$\dfrac{y_3}{-7176 + 10590} = \dfrac{y_5}{-10590 + 23920} = \dfrac{1}{300 - 90}$

$\dfrac{y_3}{3414} = \dfrac{y_5}{13330} = \dfrac{1}{210}$

$y_3 = \dfrac{3414}{210}$; $y_5 = \dfrac{13330}{210}$

$y_3 = 16.2571$ $y_5 = 6.33$ 63.47

31. Fit a polynomial of degree 3 and hence determine $y(3.5)$ for the following data.

x:	3	4	5	6
y:	6	24	60	120

Difference table.

x	y	1st	2nd	3rd
3	6			
		18		
4	24		18	
		36		6
5	60		24	
		60		
6	120			

By Newton's forward Interpolation formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-3}{1} = x-3$$

$x = x \quad x_0 = 3 \quad h = 1$

$$y(3.5) = 6 + (x-3)\cdot 18 + \frac{(x-3)(x-3-1)}{2!}18 + \frac{(x-3)(x-3-1)(x-3-2)}{3!}6$$

$$= 6 + 18x - 54 + \frac{(x-3)(x-4)}{\cancel{2}} \times \cancel{18}^{9} + \frac{(x-3)(x-4)(x-5)}{\cancel{6}} \times \cancel{6}$$

$$= 6 + 18x - 54 + (x^2 - 3x - 4x + 12)9 + [x^2 - 3x - 4x + 12](x-5)$$

$$= \cancel{6} + 18x - \cancel{54} + 9x^2 - 27x - 36x + \cancel{108} + x^3 - 3x^2 - 4x^2 + 12x - 5x^2 + 15x + 20x - \cancel{60}$$

$$= x^3 - 3x^2 + 2x$$

put x 3.5

$$y(3.5) = (3.5)^3 - 3(3.5)^2 + 2(3.5)$$

$$= 42.875 - 3(12.25) + 7$$

$$= 42.875 - 36.75 + 7$$

$$\therefore y(3.5) = 13.125$$

32. Find the cubic polynomial which takes the following values.

$y(0) = 1, \; y(1) = 0, \; y(2) = 1, \; y(3) = 10.$

Hence obtain $y(4)$

$y(0)=1, \quad y(1)=0 \quad y(2)=1. \quad y(3)=10$

Difference table

x	y	1st	2nd	3rd
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

Newtons forward interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{X-X_0}{h} = \frac{x-0}{1} = x$$

$X=x \quad X_0=0, \quad h=1$

$$y_n = 1 + x(-1) + \frac{x(x-1)\cancel{2}}{\cancel{2}} + \frac{x(x-1)(x-2)}{\cancel{6}}\cancel{6}$$

$$= 1 - x + x^2 - x + (x^2-x)(x-2)$$

$$= 1 - x + x^2 \cancel{-x} + x^3 - x^2 - 2x^2 \cancel{+2x}$$

$$= x^3 - 2x^2 + 1$$

put $x=4$

$$y(4) = 4^3 - 2(4)^2 + 1$$

$$= 64 - 32 + 1$$

$\therefore y(4) = 33$ [0, 3] interval '4' is out of intravel so it is called extrapolation.

33 find the polynomial interpolating the data

x : 0 1 2

y : 0 5 2 Difference Table

x	y	1st	2nd
0	0		
		5	
1	5		-8
		-3	
2	2		

Newton's forward Interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$x = x \quad x_0 = 0 \quad h = 1$

$$[y_n = x + x(0) + \frac{x(x-1)}{2}5 + \frac{x(x-1)(x-2)}{\cancel{6}3}(\cancel{-8}4)$$

$$= x + 0 + \frac{(x^2-x)5}{2} + \frac{(x^2-x)(x-2)}{3}(-4)$$

$$= x + \frac{5x^2-5x}{2} + \frac{(x^3-x^2-2x^2+2x)}{3}(-4)$$

$$= x + 5x^2 - 5x]$$

$$y_n = 0 + x.5 + \frac{x(x-1)}{\cancel{2}}x(\cancel{-8})4$$

$$= 5x - (x^2-x)4$$

$$= 5x - 4x^2 + 4x$$

$$\therefore y_n = -4x^2 + 9x$$

34. Find the polynomial of deg(4) which takes the following Values

x :	2	4	6	8	10
y :	0	0	9	0	0

35. Use Newtons Forward Difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data

x :	1	2	3	4
y :	26	18	4	1

and find and $x = 5$

solul Form the Difference table

35.

x	y	1st	2nd	3rd
1	26			
		-8		
2	18		-6	
		-14		16
3	4		11	
		-3		
4	1			

17 Mistake Practice well

$$\begin{array}{r} 26 \\ 18 \\ \hline 8 \end{array}$$

From Newton's Interpolation forward formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-1}{1} = x-1$$

$x = x;\ x_0 = 1;\ h = 1$

$$y_n = 26 + (x-1)(-8) + \frac{(x-1)(x-1-1)}{2}(-6)^{3} + \frac{(x-1)(x-1-1)(x-2-1)}{\cancel{6}\,3} 8$$

$$= 26 - 8x + 8 - (3x-3)(x-1-1) + \frac{(x-1)(x-2)(x-3)}{3}8$$

$$= 26 - 8x + 8 - (3x-3)(x-2) + \frac{[x^2 - x - 2x + 2](x-3)}{3} \times 8$$

$$= 26 - 8x + 8 - [3x^2 - 3x - 6x + 6] + [x^3 - x^2 - 2x^2 + 2x - 3x^2 + 3x + 6x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 78 - 24x + 24 - 9x^2 + 27x - 18 + (x^3 - 6x^2 + 11x - 6)8$$

$$= 84 - 24x - 9x^2 + 27x + 8x^3 - 48x^2 + 88x - 48$$

$$y_n = 8x^3 - 57x^2 + 91x + 36$$

$26 \times 3 = 78$; $84 - 48 = 36$; $78 + 24 = 102$; $102 - 18$

put $x = 5$

$$y(5) = 8(5)^3 - 57(5)^2 + 91(5) + 36$$

$$= 8(125) - 57(25) + 455 + 36$$

$$= 1000 - 1425 + 455 + 36$$

$$\therefore y(5) = 66$$

34. Forming the difference table

x	y	1st	2nd	3rd	4th
2	0				
		0			
4	0		1		
		1		-3	
6	1		-2		+6
		-1		3	
8	0		1		
		0			
10	0				

From Newtons Forward interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$n = \frac{x-x_0}{h} = \frac{x-2}{2}$$

$$y_n = 0 + \frac{x-2}{2}\,0 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)}{2}\,1 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)}{\cancel{6}_2}(-3)$$

$$+ \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)\left(\frac{x-2}{2}-3\right)}{\cancel{24}_4}\cancel{6}$$

$$y_n = \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)}{2} - \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{2} + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)\left(\frac{x-8}{2}\right)}{4}$$

$$y_n = \frac{(x-2)(x-4)}{8} - \frac{(x-2)(x-4)(x-6)}{16} + \frac{(x-2)(x-4)(x-6)(x-8)}{64}$$

$$y_n = \frac{x^2-2x-4x+8}{8} - \frac{[x^2-2x-4x+8][x-6]}{16} + \frac{[x^2-2x-4x+8][x^2-6x-8x+48]}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-2x^2-4x^2+8x-6x^2+12x+24x-48]}{16}$$

$$+ \frac{x^4-6x^3-8x^3+48x^2-2x^3+12x+16x^2-96x-4x^3+24x^2+32x^2+192x+8x^2-48x-64x+404}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-12x^2+44x-48]}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{x^3-6x+8}{8} - \frac{x^3+12x^2-44x+48}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{8x^3-48x+64-4x^3+48x^2-176x+192+x^4-20x^3+44x+404}{64}$$

$$y_n = x^4-16x^3+48x^2-180x+666 \;//$$

Date 10/7/18 i) find the no. of students from the following data who secured marks not more than 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	35 +	48 +	70 +	40 +	22

Difference table

Marks (x) (below)	No. of students (y)	1st	2nd	3rd	4th
40	35				
		48			
50	83		22		
		70		-52	
60	153		-30		64
		40		12	
70	193		-18		
		22			
80	215				

From Newton's forward interpolation formula.

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$n = \frac{x - x_0}{h}$; $x = 45$, $x_0 = 40$, $h = 10$ why we took 40 instead of 30

$$n = \frac{45-40}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$y(45) = 35 + (0.5)(48) + \frac{(0.5)(0.5-1)}{2!}\,22 + \frac{(0.5)(0.5-1)(0.5-2)}{6_3} \times (-52)_{26} + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24_3} \times 64_8$$

$$y(45) = 35 + 24 - 2.75 + \frac{(0.5)(-0.5)(-1.5)(-26)}{3} + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{3} \times 8$$

$$y(45) = 35 + 24 - 2.75 - 3.25 - 2.5$$

$\therefore$ $y(45) = 50.5$

$\therefore$ No. of students who secured below 45 marks $= 50.5$
$= 51$ (approximate)

ii) No.of students in between 40 and 45 =
No.of students secured 45 marks - No.of students secured 40 marks

$= 51-35$

$= 16$

above 45

$= 215-51$

$= 164$

3) find the no.of men getting the wages between Rs.10 and Rs.15 from the following table

wages	0-10	10-20	20-30	30-40
Frequency	9 +	30 +	35 +	42

Sol) Difference Table

x (below)	y	1st	2nd	3rd
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

From Newtons Forward interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

$n = \frac{x - x_0}{h}$ $\quad x = 15$; $x_0 = 10$; $h = 10$

$n = \frac{15-10}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$

$$y(15) = 9 + 30(0.5) + \frac{(0.5)(0.5-1)}{2} 5 + \frac{(0.5)(0.5-1)(0.5-2)}{\cancel{6}\,3} \cancel{2}$$

$$= 9 + 15.0 + \frac{(0.5)(-0.5)}{2}.5 + \frac{(0.5)(-0.5)(-1.5)}{3}$$

$$= 9 + 15 - 0.625 + 0.125$$

$y(15) = 23.5$

$\therefore$ No.of men got the wages below Rs.15 = 23.5

= 24 (approximately)

The wages in between Rs.10 and Rs.15

No.of men who got below Rs.15 - below Rs.10

$= 24 - 9 = 15$

$$= 9 + 42 + 210 + 105 \quad = 366$$

40 Using Newtons Backward interpolation formula, find $e^{-1.9}$ from the following table

x :	1	1.25	1.5	1.75	2
$y = e^{-x}$:	0.3679	0.2865	0.2231	0.1738	0.1353

Sol: Difference table

x	$y = e^{-x}$	1st	2nd	3rd	4th
1	0.3679	~~-0.0321~~ -0.0814	0.018 ~~0.5687~~	-0.0037	0.0006
1.25	0.2865	-0.0634	0.0141 ~~0.1127~~	-0.0033	
1.5	0.2231	-0.0493	0.0108		
1.75	0.1738	-0.0385			
2	0.1353				

From Newton's Backward Interpolation formula

$$y_n = y_n + n \nabla y_n + \frac{n(n+1)}{2!} \nabla^2 y_n + \frac{n(n+1)(n+2)}{3!} \nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}$$

$[y_n = 0.1353 + \,]$ $\quad n = \dfrac{x - x_0}{h}$ $\quad h = 0.25$; $x = 1.9$; $x_0 = 2$

$$n = \frac{1.9 - 2}{0.25} = \frac{-0.1}{0.25} = -0.4$$

$$y_{1.9} = 0.1353 + (-0.4)(-0.0385) + \frac{(-0.4)(-0.4+1)(0.0108)}{1.2}$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)}{1.2.3}(-0.0033) + \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{1.2.3.4} \times 0.0006$$

$$[y_n = 0.1353 + 0.0154 + 6.264 \times 10^{-3} - 8.448 \times 10^{-4}$$

$$= 3.6756]$$

$y_{7.9} = 0.1353 + 0.0154 - 0.00\frac{934}{1296} + 0.0002112 + 0.00002496$

$y_{7.9} = 0.13797614 = 0.138$

41. Find the $\cos(25°)$ and $\cos(75°)$ from the following data.

H.W

x	10	20	30	40	50	60	70	80
$y = \cos x$	0.9848	0.9397	0.866	0.766	0.6428	0.5	0.3420	0.1727

42. Using Newton's backward formulae find the value of y and $x = 36$ from the following data.

x :	21	25	29	33	37
y :	18.4	17.8	17.1	16.3	15.5

Solu) 41.

x	y	1st	2nd	3rd	4th	5th	6th	7th
10	0.9848							
		-0.0451						
20	0.9397		-0.0286					
		-0.0737		0.0023				
30	0.866		-0.0263		0.0008			
		-0.1		0.0031		-0.003		
40	0.766		-0.0232		0.0005		0.0006	
		-0.1232		0.0036		0.0003		-0.0016
50	0.6428		-0.0196		0.0008		-0.001	
		-0.1428		0.0044		-0.0013		
60	0.5		-0.0152		-0.0005			
		-0.158		0.0039				
70	0.3420		-0.0113					
		-0.1693						
80	0.1727							

Newton's Forward Interpolation Formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 y_0 + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}\Delta^6 y_0$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{7!}\Delta^7 y_0$$

$n = \frac{x - x_0}{h}$; $x = 25$; $x_0 = 10$; $h = 10$ $\quad n = \frac{25-10}{10} = \frac{15}{10} = 1.5$

$$y_n = 0.9848 + 1.5(-0.0451) + \frac{1.5(1.5-1)}{2}(-0.0286) + \frac{(1.5)(1.5-1)(1.5-2)}{6}0.002$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)}{24}0.0008 + \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)}{120}(-0.0003)$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)}{720}(0.0006)$$

$$+ (1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)/5040\ (-0.0016)$$

$y_n = 0.9848 - 0.06765 - \frac{0.02145}{2} - 0.0625 \times 0.0023$
$+ 0.0234375 \times 0.0008 + 7.8125 \times 10^{-4} \times 0.0003 +$
$+ 6.510416667 \times 10^{-5} \times 0.0006 + 4.650297619 \times 10^{-6} \times 0.0016$

$y_n = 0.9848 - 0.06765 - 0.010725 - 0.00014375 + 0.00001875 + 0.000000007440047619$
$+ 0.000000234375 + 0.0000000390625$

$y(\cos 25) = 0.9063002809$

Newtons Backward Interpolation formulae.

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)}{5!}\nabla^5 y_n + \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)}{6!}\nabla^6 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}{7!}\nabla^7 y_n$$

$n = \frac{x - x_0}{h}$; $x = 75$, $x_0 = 80$; $h = 10$ $\quad n = \frac{75-80}{10} = \frac{-5}{10} = -0.5$

$$y_n = \cancel{-0.5}\; 0.9848 + (-0.5)(-0.0451) + \frac{(-0.5)(-0.5+1)}{2!}(-0.0286)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{6} \times 0.0023 + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{24}\, 0.0008$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{120}(-0.0003)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{720} \times 0.0006$$

wrong within the brackets

$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)(-0.5+6)}{5040}(-0.0016)$$

$$y_n = 0.9848 + 0.02255 + \frac{0.00715}{2} + \frac{0.0008625}{6} - \frac{0.00075}{24} + \frac{0.00098437}{120}$$
$$- \frac{0.008859375}{720} + \frac{0.1299375}{5040}$$

$\cos(75) = 0.9848 + 0.02255 + 0.003575 + 0.00014375 - 0.00003125$
$+ 0.000008203125 - 0.0000123046875 + 0.00002578125$

$\cos(75) = 1.01105918$

$$= 0.1727 + (-0.5)(-0.1693) + \frac{(-0.5)(-0.5+1)}{2}(-0.0113)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.0039) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(0.0005)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}(-0.0013)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{6!}(-0.001)$$

Date 13/7/18

Legranges Interpolation Formula

Consider $y=f(x)$ be the given function. x takes the values $x_0, x_1, x_2, x_3, x_4, ----$ the corresponding y values are $y_0, y_1, y_2, y_3, y_4, -----$ respectively. Then

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

Date 14/7/18

1. Using Legranges formula to find $f(6)$ from the following table.

	x_0	x_1	x_2	x_3	x_4
x	2	5	7	10	12
$f(x)$	18	180	448	1210	2028
	y_0	y_1	y_2	y_3	y_4

By Legranges interpolation formula

solu)

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$y(6) = \frac{(6-5)(6-7)(6-10)(6-12)}{(2-5)(2-7)(2-10)(2-12)} \cdot 18 + \frac{(6-2)(6-7)(6-10)(6-12)}{(5-2)(5-7)(5-10)(5-12)} \times 180$$

$$+ \frac{(6-2)(6-5)(6-10)(6-12)}{(7-2)(7-5)(7-10)(7-12)} \times 448 + \frac{(6-2)(6-5)(6-7)(6-12)}{(10-2)(10-5)(10-7)(10-12)} \times 1210$$

$$+ \frac{(6-2)(6-5)(6-7)(6-10)}{(12-2)(12-5)(12-7)(12-10)} \times 2028$$

$$y(6) = \frac{1(-1)(-4)(-6)}{(-3)(-5)(-8)(-10)} \times 18 + \frac{4(-1)(-4)(-6)}{3(-2)(-5)(-7)} \times 180$$

$$+ \frac{4(1)(-4)(-6)}{5(2)(-3)(-5)} \times 448 + \frac{(4)(1)(-1)(-6)}{8 \cdot 5 \cdot 3 \cdot 2} \times 1210 + \frac{4 \cdot 1 \cdot (-1)(-4)}{10 \cdot 7 \cdot 5 \cdot 2}$$

$$y(6) = \frac{-\cancel{24}}{\cancel{1200}\ 25} \times 189 - \frac{\cancel{72}\ 36}{-\cancel{210}\ 15} 180 + \frac{72}{150} \times 448 - \frac{24}{\cancel{240}\ 10} \times 1210 + \frac{16}{700} \times 2028$$

$$y(6) = \frac{-9}{25} + \frac{6480}{15} + \frac{72}{150} \times 448 - 121 + \frac{16 \times 2028}{700}$$

$$y(6) = -0.36 + 432 + 215 - 121 + 46.354$$

$$y(6) = 572.714$$

$$y(6) = \frac{-9}{25} + \frac{576}{7} + \frac{7168}{25} - 121 + \frac{8112}{175}$$

$$y(6) = -121 + \frac{-9+7168}{25} + \frac{576}{7} + \frac{8112}{175}$$

$$y(6) = -121 + \frac{7159}{25} + \frac{576}{7} + \frac{8112}{125}$$

$$y(6) = -121 + 286.36 + 82.2857 + 46.3542$$

$$\therefore y(6) = 293.9999 = 294$$

2. Using the Legranges interpolation formulae find the value of $y(10)$ from the following table

x:	5_{x_0}	6_{x_1}	9_{x_2}	11_{x_3}
y:	12_{y_0}	13_{y_1}	14_{y_2}	16_{y_3}

Solu) By Legranges interpolation formulae

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x_1-x_2)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} 13$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} .16$$

$$y(10) = \frac{\cancel{4}.1.(-1)}{(-1)(-\cancel{4})(-\cancel{6})} \cdot \cancel{12}\,2 + \frac{5.1.(-1)}{1(-3)(-5)} 13 + \frac{5(4)(-1)}{4(3)(-2)} \cancel{14}\,7$$

$$+ \frac{5(4)(1)}{6.5.2} \cdot \cancel{16}\,8$$

$$y(10) = \frac{+2}{\cancel{2}} - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = \frac{6-13+35+16}{3}$$

180

1210

2028

$y(10) = \frac{44}{3}$

$\therefore y(10) = 14.667$

3. find the Cubic Legranges Interpolating polynomial from the following data.

x :	0 x_0	1 x_1	2 x_2	5 x_3
$f(x)$:	2 y_0	3 y_1	12 y_2	147 y_3

solu) The Legranges Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} 2 + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} 3$$

$$+ \frac{(x-0)(x-2)(x-5)}{(2-0)(2-1)(2-5)} 12 + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} 147$$

$$= \frac{(x-1)(x-2)(x-5)}{(-1)(-2)(-5)} \cancel{2} + \frac{x(x-2)(x-5)}{1(-1)(-4)} 3$$

$$+ \frac{x(x-1)(x-5)}{2 \cdot 1 \cdot (-3)} \cancel{12}^{2} + \frac{x(x-1)(x-2)}{5(4)(3)} 147$$

$$= -\frac{(x-1)(x-2)(x-5)}{5} + \frac{x(x-2)(x-5)}{4} \times 3$$

$$- x(x-1)(x-5) \times 2 + \frac{x(x-1)(x-2)}{60} \times 147$$

$$= -\frac{(x^2-x-2x+2)(x-5)}{5} + \frac{(x^2-2x)(x-5)}{4} \times 3$$

$$- (x^2-x)(x-5) 2 + \frac{(x^2-x)(x-2)}{60} \times 147$$

$$= -[x^3-x^2-2x^2+2x-5x^2+5x+10x-10]/5$$

$$+ \frac{[x^3-2x^2-5x^2+10x]3}{4} - [x^3-x^2-5x^2+5x]2$$

$$+ \frac{[x^3-x^2-2x^2+2x] \times 147}{60}$$

$$= -[x^3 - 7x^2 + 10x]$$

$$= -\frac{[x^3-8x^2+17x-10]}{5} + \frac{[x^3-7x^2+10x]3}{4} - \frac{[x^3-6x^2+5x]}{2}$$

$$+ \frac{[x^3-3x^2+2x]\,49}{20}$$

$$= \frac{-x^3+8x^2-17x+10}{5} + \frac{3x^3-21x^2+30x}{4} - 2x^3+12x^2-10x$$

$$+x^3-3x^2+2x + \frac{49x^3-147x^2+98x}{20}$$

$$= \frac{-4x^3+32x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3-147x^2+98x}{20}$$

$$= \frac{20x^3+20x^2-20x+40}{20}$$

$$= \frac{20(x^3+x^2-x+20)}{20}$$

$$\therefore f(x) = x^3+x^2-x+20$$

4. find the Legranges interpolating polynomial for the given data:

x:	1 x_0	2 x_1	3 x_2	4 x_3
$f(x)$:	1 y_0	8 y_1	27 y_2	64 y_3

Solu) The Legranges interpolation formulae.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$f(x) = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}1 + \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)}8$$

$$+ \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)}27 + \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}64$$

$$f(x) = \frac{(x-2)(x-3)(x-4)}{(-1)(-2)(-3)} + \frac{(x-1)(x-3)(x-4)}{1(-1)(-2)}84$$

$$+ \frac{(x-1)(x-2)(x-4)}{2(1)(-1)}27 + \frac{(x-1)(x-2)(x-3)}{3(2)(1)}\cancel{64}\,32$$

$$= \frac{(x^2-2x-3x+6)(x-4)}{-6} + \frac{(x^2-x-3x+3)(x-4)}{1}4$$

$$+ \frac{(x^2-x-2x+2)(x-4)}{-2}27 + \frac{[x^2-x-2x+2](x-3)}{3}32$$

$= -\frac{(x^2-5x+6)(x-4)}{6} + (x^2-4x+3)(x-3)\ \cancel{32}$

$- \frac{(x^2-3x+2)(x-4)}{2}\, 27 + \frac{[x^2-3x+2][x-3]}{3}\, 32$

$[= -\frac{[x^3-5x^2+6x-4x^2+20x-24]}{6} + [x^3-4x^2+3x-3x^2+12x+9]\,4$

$- \frac{[x^3-3x^2+2x-4x^2+12x-8]}{2}\, 27 + \frac{[x^3-3x^2+2x-3x^2+9x-6]}{3}\, 32$

$= \frac{-x^3+5x^2-6x+4x^2-20x+24}{6} + 4x^3-16x^2+12x-12x^2+48x+36$

$- \frac{27x^3+81x^2-54x+108x^2-324x+216}{2} + \frac{32x^3-96x^2+64x-96x^2+288x-192}{3}$

wrong

$= -x^3+5x^2-6x+4x^2-20x+24+24x^3-96x^2+72x-72x^2+$
$\frac{288x+216-81x^3+243x^2-162x+324x^2-972x+648+64x^3-192x^2+128x-192x^2+576x-384}{6}$

$= 6x^3]$

$= -[x^3-7x^2+12x-2x^2+14x-24]+24[x^3-7x^2+12x-x^2+7x-12]$
$\frac{-27\times3(x^3-6x^2+8x-x^2+6x-8]+64(x^3-5x^2+6x-x^2+5x-6)}{6}$

$= \frac{-[x^3-9x^2+26x-24]+24[x^3-8x^2+19x-12]-27[x^3-7x^2+14x-8]+64[x^3-6x^2+11x-6]}{6}$

$= \frac{+[x^3+9x^2-26x+24]+24x^3-192x^2+456x-288-81x^3+567x^2-1134x+648+64x^3-384x^2+704x-384}{6}$

$= \frac{1}{6}[6x^3+0+0+0] = x^3$

5. Using Lagranges Interpolation formula to fit a polynomial to the following data

	x_0	x_1	x_2	x_3
x :	-1	0	2	3
	u_0	u_1	u_2	u_3
u_x :	-8	3	1	12

And also find the value u_1

Sol: By Lagranges interpolation formulae

$$u_x = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} u_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} u_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} u_3$$

$$u_x = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)} 3$$

$$+ \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)} 1 + \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)} 12$$

$$[u_x = \frac{(x-0)(x-2)(x-3)}{-1 \times -3 \times +4}(+\cancel{8}^{2}) + \frac{(x+1)(x-2)(x-3)}{-2 \times \cancel{3} \times 1} 12 \cancel{3}$$

$$+ \frac{(x+1)x(x-3)}{3 \cdot 2(-1)} + \frac{(x+1)(x)(x-2)}{\cancel{4} \cdot \cancel{3} \cdot 1} \cancel{12}^{3}$$

$$u_x = \frac{2x[x^2-2x-3x+6]}{3} + \frac{x[x^2+x-2x-2]}{+4}$$

wrong

$$+ \frac{[x^2+x-3x-3]x}{-6} + \frac{[x^2+x-2x-2]x}{1}$$

$$u_x = \frac{2x^3-4x^2-6x^2+12x}{3} + \frac{x^3+x^2-2x^2-2x}{4}$$

$$\frac{x^3+x^2-3x^2-3x}{6} + \frac{x^3+x^2-2x^2-2x}{}$$

$$u_x = \frac{2x^3-10x^2+12x}{3} + x^3 - 5x^2]$$

$$u_x = (1) \cdot 2x^3 \cdot \frac{x(x^2-5x+6)}{-1 \times 3 \times -4} x - 8^2 + \frac{(x+1)(x^2-5x+6)}{1 \times -2 \times -3} \times 3$$

$$+ \frac{(x+1)(x^2-3x)}{3 \times 2 \times -1} + \frac{(x+1)(x^2-2x)}{4 \times 3 \times 1} \times 12^{3}$$

$$u_x = \frac{2x^3-10x^2+12x}{3} + \frac{x^3-5x^2+6x+x^2-5x+6}{2} - \frac{(x^3-3x^2+x^2-3x)}{6}$$

$$+ (x^3-2x^2+x^2-2x)$$

$$u_x = \frac{2(2x^3-10x^2+12x) + 3(x^3-4x^2+x+6) - (x^3-3x^2+x^2-3x) + 6(x^3-2x^2-2x)}{6}$$

$$ux = \frac{1}{6}\{4x^3-20x^2+24x+3x^3-12x^2+3x+18-x^3+2x^2+3x+6x^3-6x^2-12x\}$$

If $x = 1$

$$= \frac{12x^3-36x^2+18x+18}{6} = 2x^3-6x^2+3x+3$$

$$u_1 = 2(1)^3 - 6(1)^2 + 3(1) + 3 = 2$$

Date 17/7/18

Central Differences

Gauss - Forword Interpolating Formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \cdots$$

Gauss - Backword Interpolating Formulae

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-3} + \cdots$$

1 find $f(2.5)$ using the following table.

x:	1	2	3	4
$f(x)$:	1	8	27	64

Difference table

why we used only Gauss forward why should's newton's forward

solu)

x	y	1st	2nd	3rd
$1\ x_{-1}$	$1\ y_{-1}$			
		$7\ y_{-1}$		
$2\ x_0$	$8\ y_0$		$12\ y_{-1}$	
		$19\ y_0$		$6\ y_{-1}$
$3\ x_1$	$27\ y_1$		$18\ y_0$	
		$37\ y_1$		
$4\ x_2$	$64\ y_2$			

Amma
Nanna
Chinni

Gauss forward interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1}$$

$n = \frac{x - x_0}{h}$; $x_0 = 2.0$ $x = 2.5$; $h = 1$

why we $x_0 = 2$

$n = \frac{2.5 - 2}{1} = 0.5$

$$y_n = 8 + (0.5)(19) + \frac{(0.5)(0.5-1)}{2!}12 + \frac{(0.5+1)(0.5)(0.5-1)}{3!}6$$

$$= 8 + 9.5 + \frac{(0.5)(-0.5)}{\cancel{2}}\cancel{12}^{6} + \frac{(1.5)(0.5)(-0.5)}{\cancel{2}\cancel{3}} \times \cancel{6}$$

$$= 8 + 9.5 - \cancel{5.75}\ 1.5 - 0.375$$

$y(2.5) = 15.625$

2. From the following table find y when $x = 38$

x:	30	35	40	45	50
y:	15.9	14.9	14.1	13.3	12.5

Difference table

x	y	1st	2nd	3rd	4th
$30\ x_{-1}$	$15.9\ y_{-1}$				
		$-1\ y_{-1}$			
$35\ x_0$	$14.9\ y_0$		$0.2\ y_{-1}$		
		$-0.8\ y_0$		$-0.2\ y_{-1}$	
40	$14.1\ y_1$		$0\ y_0$		$0.2\ y_{-1}$
		$-0.8\ y_1$		$0\ y_0$	
45	$13.3\ y_2$		$0\ y_1$		
		$-0.8\ y_2$			
50	$12.5\ y_3$				

By applying Gauss forword interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y$$

$$n = \frac{x - x_0}{h}\ ;\ x = 38\ \ x_0 = 35\ ;\ h = 5$$

$$n = \frac{38-35}{5} = \frac{3}{5} = 0.6$$

$$y_n = 14.9 + (0.6)(-0.8) + \frac{(0.6)(0.6-1)}{2!}(0.2) + \frac{(0.6+1)(0.6)(0.6-1)}{3!}(-0.2)$$

$$+ \frac{(0.6+1)(0.6)(0.6-1)(0.6-2)}{4!}(0.2)]$$

$$y_{(38)} = 14.9 - 0.48 - 0.024 + 0.0128 + [0.00]$$

$$y_{(38)} = 14.4088$$

3. Using Gauss forword interpolating formulae we find $f(3.3)$ from the following data

x	1	2	3	4	5
$f(x)$	15.3	15.1	15.	14.5	14

Difference table

why we used Gauss forward & Gauss backward

x	y	1st	2nd	3rd	4th
$1\ x_{-2}$	$15.3\ y_{-2}$	$-0.2\ y_{-2}$	$-0.9\ y_{-2}$	$-0.5\ y_{-2}$	-2.5
$2\ x_{-1}$	$15.1\ y_{-1}$				$0.9\ y_{-2}$
		$-0.1\ y_{-1}$			
$3\ x_0$	$15\ y_0$		$-0.4\ y_{-1}$	$+0.4\ y_{-1}$	
		$+0.5\ y_0$			
4	14.5		$-0.0\ y_0$		
5	14	-0.5			

By applying Gauss forward interpolating formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$n = \frac{x - x_0}{h}$ $x = 3.3$; $x_0 = 3$; $h = 1$; $n = \frac{3.3 - 3}{1} = 0.3$

$$y(3.3) = 15 + 0.3(-0.5) + \frac{(0.3)(0.3-1)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(0.4)$$

$$+ \frac{(0.3+1)(0.3)(0.3-1)(0.3-2)}{4!} \times 0.9$$

$$y(3.3) = 15 - 0.15 + 0.042 - 0.0286 + 0.02016] - 0.0182 + 0.0580125$$

$y(3.3) = 14.88656 \quad 14.9318125$

$= 15 - 0.15 + 0.042 - 0.182 + 0.01740375 = 14.89120375$

4. find the polynomial which quit the data in the following table using Newtons forward formula

x	3	5	7	9	11
y	6	24	58	108	174

solu) Difference table

x	y	1st	2nd	3rd	4th
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	108		16		
		66			
11	174				

By applying Newtons forward formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \cdots$$

$n = \frac{x - x_0}{h}$ $x_0 = 3$; $x_n = x$ $h = 2$ $n = \frac{x-3}{2}$

$n = \frac{x-3}{2}$

$$y_n = 6 + \left(\frac{x-3}{\cancel{2}}\right)\cancel{18}^{9} + \frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2} - 1\right)}{\cancel{2}}\cancel{16}^{8}$$

$$= 6 + 9x - 27 + \left(\frac{x-3}{\cancel{2}}\right)\left(\frac{x-5}{\cancel{2}}\right) \times \cancel{16}\,\cancel{8}\,4\,2\,\frac{1}{\cancel{2}}$$

$= 6 + 9x - 27 + [x-3][x-5]2$

$= 6 + 9x - 27 + [x^2 - 3x - 5x + 15]2$

$= -21 + 9x + 2x^2 - 6x - 10x + 30$

$y_n = 2x^2 - 7x + 9$

Date 7/8/18 By using Gauss Backward interpolating formulae find the value of y and x = 3.3 from the following data

x	1	2	3	4	5
y	15.3	15.1	15.	14.5	14

Solu) Difference table

x	y	1st	2nd	3rd	4th
1 x_{-2}	15.3 y_{-2}	-0.2 y_{-2}	-0.2 y_{-2}	-0.5 y_{-2}	0.9 y_{-2}
2 x_{-1}	15.1 y_{-1}	-0.1 y_{-1}	-0.4 y_{-1}	$+0.4$ y_{-1}	
3 x_0	15 y_0	0.5 y_0	0 y_0		
4	14.5	-0.5			
5	14				

By using ~~Newton's~~ Gauss Backword formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y$$

$$n = \frac{x - x_0}{h} \ ; \quad x = 3.3 \ ; \ x_0 = 3 \ ; \quad h = 1 \ ; \ n = \frac{3.3 - 3}{1} = 0.3$$

$$y(3.3) = 15 + (0.3)(0.1) + \frac{(0.3+1)(0.3)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(-0.5)$$

$$+ \frac{(0.3+2)(0.3+1)(0.3)(0.3-1)}{4!}(+0.9)$$

$[y_n = 15 + 4.53 - 0.0195 + 0.0182]$

$y(3.3) = 15 - 0.03 - 0.078 + 0.02275 - 0.02354625$

$y(3.3) = 14.89120375$

$y(3.3) = 14.8912$

6 From the following table find the value of y when x = 1.35

x:	1	1.2	1.4	1.6	1.8	2
y:	0.0	-0.112	-0.016	0.336	0.992	2

why we used Gauss backward.

x	y	1st	2nd	3rd	4th	
1	0.0					
		$\underline{-0.112}\ \Delta y_{-1}$				
$\underline{1.2}\ x_0$	$\underline{-0.112}\ y_0$		$\underline{0.208}\ \Delta^2 y_{-1}$			
		0.096		$\underline{0.048}\ \Delta^3 y_{-1}$		
1.4	-0.016		0.256		0	
		0.352		0.048		0
1.6	0.336		0.304		0	
		0.656		0.048		
1.8	0.992		0.352			
		-0.992				
2	2					

By applying Gauss backward interpolating formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2}$$

$n = \frac{x - x_0}{h}$ $\quad x = 1.35 \quad x_0 = 1.2 \ ; \ h = 0.2 \ ; \ n = \frac{1.35 - 1.2}{0.2} = 0.75$

$$y(1.35) = (-0.112) + (0.75)(-0.112) + \frac{(0.75)(0.75+1)}{2!}(0.208)$$

$$= -0.112 - 0.084 + 0.1365$$

$$\therefore y(1.35) = -0.0595$$